Lifting under-achievement in Mathematics

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Sabbatical Study 2015
I met personally with:

Charles Lovitt  Mathematics Consultant Melbourne
Prof. Doug Clarke  Australian Catholic Uni, Melbourne
Dr Barbara Clarke  Monash University, Melbourne

I read and studied the work of:

Prof. Jo Boaler  Stanford Graduate School of Education
Dr Sue Gifford  Cambridge University
Stein & Engle et al  Pittsburg & Berkeley Universities USA
Freudenthal Institute for Science and Mathematics Education (Netherlands)
Part 1

Analysing a closed task

- What does brain science tell us
- What does Mindset Psychology tell us
- What do International Assessments tell us
- What do international math experts say
- What does Charles Lovitt (my mentor) say
A math lesson which values memorisation and steps (procedural fluency).

What does $5 + 6 =$

- a closed task - you are either right or wrong
- commonly used in work sheets to teach to the middle of a year level
Good!
You got that right ...
as a reward you can do
50 more like that on
page 56.
13?
I didn't think he would know....
Wrong!
You can join the Donkeys' Group and do 10 more like that!
I know I am no good at math ....

I am always in the Donkeys' Group
Brain Science tells us:

- traditionally people thought intelligence was innate
- we now know that no one is born with a math brain
- with good experiences the brain has the capacity to change, rewire and grow
- students can grasp high level ideas but won’t develop brain connections if they are given low level work and negative messages
recent research shows that when students make a mistake in math, their brain grows, synapses fire, and connections are made.

when students do a page of exercises correctly - there is no brain growth - the brain barely lights up.

when children struggle with a problem - their brain lights up and grows connections.
Mindset Psychology says:

People with ‘fixed mindsets’ believe....

- you are either smart or you are not
- giving children lots of exercises so they get fast at recall and memorisation will make them good at mathematics
- children who struggle need lots of simple exercises and shouldn’t do higher level math
when students with a fixed mindset make a mistake they believe they are not smart and give up - they avoid challenge and want easy work to do

when children spend their time answering discrete questions with right or wrong answers it is very difficult to develop a growth mindset
Math Experts say:

part of the problem is the desperation of many parents to advance their children in math - pushing them to higher levels - faster and sooner

mathematics is not a subject that requires fast thinking

closed problems do not teach children to think, to use logic and reasoning, or to communicate
International Assessments show:

- the lowest achieving of 13 million students on PISA tests were those who used memorisation – a set of steps to remember

- the highest achieving students were those who thought of math as a set of connected big ideas
Charles Lovitt says:

Under-achievement is caused by ...

- the narrow closed-tasks teachers have favoured
- ability grouping with its negative connotations and the way it restricts coverage of the curriculum
- teachers’ fear that problem solving ignores skills development and so they favour closed tasks
Part 2

Analysing a rich mathematical task

- What does brain science tell us
- What does Mindset Psychology tell us
- What do International Assessments tell us
- What do international math experts say
- What does Charles Lovitt (my mentor) say
The common Japanese approach to a math lesson

Teachers collaborate to prepare tasks

Work in your group to find as many solutions as you can.

In small groups try this task.

\[
\begin{align*}
\quad & + \quad \\
\hline
\quad & = 11
\end{align*}
\]
Working like a mathematician.

How many solutions are there? How do you know you have got them all?
A problem never ends... there is always more!

What if the answer was 12?

In small groups try this task.
What if the answer is ...
13?
20?
43?
100?
n?
Can you develop a rule for this?
Develop the algebra..... oops! - not so predictable

<table>
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<th>n</th>
<th>6</th>
<th>7</th>
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<th>11</th>
<th>22</th>
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<tr>
<td>11</td>
<td>12</td>
<td>13</td>
<td>20</td>
<td>43</td>
<td>100</td>
<td>n</td>
<td></td>
</tr>
</tbody>
</table>

rule (if n is even) \( n/2 + 1 \)

rule (if n is odd) \( \frac{n + 1}{2} \)
What if there are 3 numbers?

+ 

= 11
What if there are 4 numbers?
The \((n,2)\) rule when:

\[
\begin{align*}
\text{(n is even)} & \quad n/2 + 1 \\
\text{(n is odd)} & \quad \frac{n + 1}{2}
\end{align*}
\]

What is the \((n,3)\) rule?

What is the \((n,4)\) rule?

What is the \((n,x)\) rule?

This demonstrates – there is always more to a problem!
The history:

This problem was a most famous problem for 200 years.

It was finally solved by an Indian man.
Brain Science tells us:

- with the plasticity of the brain – ability and intelligence grow with effort and practice
- children’s brains light up when they are faced with a challenging problem
- a child’s brain does not light up when doing meaningless memorisation tasks
- it is working on the complex that allows the brain to develop
Mindset Psychology says:
New findings about brain development and mindset psychology are inextricably linked

- the plasticity of the brain is now well known yet ignored by the fixed mindset thinking of schools
- a child’s mindset for learning and a teacher’s mindset for teaching is critical and makes a difference
- the damaging effects of ability grouping on a child’s mindset creates failure
deliberate Mindset intervention resulted in a gain in achievement for African-Americans

think what this approach could do for Maori/Pasifika in a NZ context?

think how Winchester’s ecology of learning and multiple intelligences theory... along with recent mindset and brain science could improve learning
Ability Grouping
- reduces achievement overall
- particularly for high achieving girls who fear failure when in placed top group
- high level content is taught to only some children
- discourages children by communicating ... only some are high achievers and ability is fixed
Interestingly...

- average & below average children achieve higher levels in mixed ability classes ....

- and ..... 

- high achievers achieve the same in either setting!
Math Experts say:

- tasks that are particularly valuable are those that have a low floor and a high ceiling
- anyone can access them, but they can be taken to very high levels
- math classes have valued one type of learner - children who can memorise well and calculate fast
International Assessments show that:

- children who are taught memorisation techniques are the lowest performers in OECD's Programme for International Student Assessment (PISA) tests

- the highest performers are in the Pacific rim countries -and particularly Japan and China - where collaborative problem solving is encouraged and assessment de-emphasised
Charles Lovitt says:

Features of a good math task:

- multiple entry and multiple exit points
- the starting point is simple enough for all to enter
- has a story shell or history - a context
- no task is ever finished
- caters for the 7-year gap in classes
- it develops skills, thinking & reasoning, and communication
Another example to consider.
A typical work sheet closed task

Find the average of these 3 numbers?

11, 12, 13

\[
\begin{align*}
11 & \\
12 & \\
13 & \\
\hline
36 & \\
\end{align*}
\]

\[= \frac{36}{3}\]

\[= 12\]
Open Ended approach

Find 3 numbers that make an average of 12?
- How many ways are there?
- How do you know you have got them all?

In small groups try this task.
Answer:
There are 127 ways to make an average of 12

The task is....

- open ended
- all children can enter
- ‘what if’ can be applied
- plenty of skill practice provided
An interesting dilemma

Many schools introducing MLE’s are developing the concept of ‘student agency’.

- teachers make their learning intentions (specific objectives) visible on the wall
- children rank themselves against each objective on the wall in a public display
- teachers plan workshops to meet indicated needs
- children opt in to workshops (children driving their learning = student agency)
this model suits the ‘numeracy type’ programme where learning is broken into small measurable segments

but... numeracy advisers are now promoting the problem solving approach ...

...and schools who want ‘student agency’ are finding it doesn’t fit.....

..... because problem solving is more holistic, it can’t be broken into bits when children are working like mathematicians (see next slide for details)

learning includes ....skills, logic & reasoning , and communication - not just procedural fluency
Working Mathematically
Learning to Work like a Mathematician

First give me an interesting problem.

When mathematicians become interested in a problem they:
- Play with the problem to collect & organise data about it.
- Discuss & record notes and diagrams.
- Seek & see patterns or connections in the organised data.
- Make & test hypotheses based on the patterns or connections.
- Look in their strategy toolbox for problem solving strategies which could help.
- Look in their skill toolbox for mathematical skills which could help.
- Check their answer and think about what else they can learn from it.
- Publish their results.

Questions which help mathematicians learn more are:
- Can I check this another way?
- What happens if …?
- How many solutions are there?
- How will I know when I have found them all?

When mathematicians have a problem they:
- Read & understand the problem.
- Plan a strategy to start the problem.
- Carry out their plan.
- Check the result.

A mathematician's strategy toolbox includes:
- Do I know a similar problem?
- Guess, check and improve
- Try a simpler problem
- Write an equation
- Make a list or table
- Work backwards
- Break the problem into smaller parts
- Act it out
- Draw a picture or graph
- Make a model
- Look for a pattern
- Try all possibilities
- Seek an exception
- ...

If one way doesn't work I just start again another way.
Snippets from the experts:

- the on-going influence of the Cockcroft Report
- what Trends in International Mathematics and Science Study (TIMSS) tells us
- Japan once again points the way
- some advice to teachers
Cockcroft Report UK

- this famous report was written by Professor Cockcroft following an investigation into the failure of mathematics teaching in the UK

- it continues to be highly regarded by academics
In the now-famous Paragraph 243 = \((3^5)\) of the Report Cockcroft recommended that mathematics teaching at all levels should include opportunities for:

- exposition by the teacher
- discussion between teacher and students, and between students themselves
- appropriate practical work
consolidation and practice of fundamental skills and routines

problem solving, including the application of mathematics to everyday situations

investigational work
Cockcroft reported that in any given class level there is a 7-year gap that teachers have to cater for.

5.5 yrs ...... 9 year old class ...... 12.5 yrs

Typically work sheets teach to the middle
this bores bright children
leaves slow children behind

so ... to achieve this we need problem solving and open-ended investigations
Cockcroft identified the mathematical needs of adults as being:

- ability to read numbers
- to count
- to tell the time
- to pay for purchases and give change
- to understand timetables, simple graphs and charts
- perform sensible estimations and approximations
A study of countries success in Trends in International Mathematics and Science Study (TIMSS) - especially Japan & China found…

- a lesson focuses around a single problem
- teacher is a knowledgeable guide rather than dispenser of information
- children encouraged to explain and evaluate solutions
- lessons focus on problem solving rather than mastery of facts and procedures
Japan has a strong belief that...

...children should not be subjected to the measuring of their capabilities or aptitudes....

...nor the subsequent remediation or acceleration during their 9 years of compulsory education...
Teachers need to:

- stop frequent timed testing
- make children feel good about mistakes & comfortable with struggle
- replace grades with constant feedback
- de-emphasise speed so students can think slowly and deeply
- remove groupings that transmit ‘fixed mindset’ messages
https://www.youtube.com/watch?v=4Xi6pLo58g4
Task:

In small groups summarise on a chart the key messages of this presentation and beside each message list possible actions we can take to succeed at it!